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Note on instability of compressible jets and wakes to long-wave disturbances

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A two-dimensional jet or wake is observed in a frame of reference moving with the fluid at infinity, so that the velocity $w(y)$ in the x-direction tends to zero as $y \to \pm\infty$. The fluid is assumed to be an inviscid perfect gas, to undergo adiabatic changes, and the local speed of sound to be a function of y such that $a(y) \to a_\infty$ as $y \to \pm\infty$. If the disturbance pressure has the form

$$p(y)\exp\{i\alpha(x-ct)\},$$

then the linearized equations for the disturbance may be reduced to a single equation for p (cf. Lighthill 1950, equation (7)), namely

$$(p'/m^2)' + \alpha^2(1-m^{-2})\,p = 0, \tag{1}$$

where

$$m(y) = \{w(y)-c\}/a(y). \tag{2}$$

For small wave-numbers α, solutions can be found in the manner of Drazin & Howard (1962) by expanding $p(y)$ as a series in α of the form

$$p(y) = \begin{cases} \exp\{-\alpha(1-c^2/a_\infty^2)^{\frac{1}{2}}y\}\,[1+\alpha p_1(y)+\alpha^2 p_2(y)+\ldots] & (y>0), \\ \exp\{\alpha(1-c^2/a_\infty^2)^{\frac{1}{2}}y\}\,[1+\alpha p_{-1}(y)+\alpha^2 p_{-2}(y)+\ldots] & (y<0). \end{cases} \tag{3}$$

An eigenvalue, c, can be found which reduces in the incompressible limit to the value (2·10) given in Drazin & Howard's paper. The value for a compressible jet or wake is given by

$$c/a_\infty \sim i\alpha^{\frac{1}{2}}\left\{\frac{1}{2}\int_{-\infty}^{\infty}[w(y)/a(y)]^2\,dy\right\}^{\frac{1}{2}} \quad \text{as} \quad \alpha \to 0. \tag{4}$$

The result shows that the amplification rate of this long-wave disturbance is *smaller* for a *hot* jet or wake than it is for a cold one, since $a(y)/a_\infty$ is larger in the former case.

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